**Stewart’s Theorem**

Stewart’s Theorem, though outside the Hong Kong Exam syllabus, is one of the important theorems in geometry. It is not too difficult to learn. See whether you can manage.



Given any triangle . is any arbitrary line. The base of the triangle: .

**(a)** Prove Stewart’s Theorem: by:

 **(i)** Pythagoras Theorem

 **(ii)** using trigonometry.

**(b)**

 In the diagram on the right,

 bisects .

 Find the length of .

**(c)** Prove that the sum of the square of the distances from the vertex of the right angle, in a right angled triangle, to the trisection point of the hypotenuse, is equal to the length of the hypotenuse:

**(a)** **(i)** Draw a perpendicular line from to .

 Let

 Then:



 (ii) Let

 By Cosine Law,

 Rearrange and putting we have

 Stewart’s Theorem can be re-written in the form:

 . (Mnemonic : A man and his dad put a bomb in the sink.)

**(b)** Let

 Since

 (This is angle bisector theorem: . )

 By Stewart’s Theorem

 (Taking positive root.)

**(c)** Apply Stewart theorem to in two ways, using and as the lines of cutting the triangle:

 Adding, we have

By Pythagoras Theorem,

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